

New York State Mathematics Association of Two-Year Colleges

Math League Contest ~ Fall 2022

Directions: You have one hour to take this test. Scrap paper is allowed. The use of calculators is NOT permitted, as well as computers, books, math tables, and notes of any kind. You are not expected to answer all the questions. However, do not spend too much time on any one problem. Four points are awarded for each correct answer, one point is deducted for each incorrect answer, and no points are awarded/deducted for blank responses. There is no partial credit. Unless otherwise indicated, answers must be given in *exact* form, i.e. in terms of fractions, radicals, π , etc.

1. Let $f(x) = \frac{x}{2^x - 1} + \frac{x}{2}$ and $g(x) = \frac{x}{2^x - 1} - \frac{x}{2}$, for all real $x \neq 0$. Which of the following statements are/is true?
- I. f is an even function. II. g is an even function. III. $f \cdot g$ is an even function.
- A) I only B) II only C) III only D) I and II only E) I and III only

2. $\frac{\frac{\frac{1}{1+1}}{1+1+1}}{\frac{1+1+1+1}{1+1+1+1}} = ?$ A) $\frac{1}{576}$ B) $\frac{1}{64}$ C) $\frac{1}{9}$ D) $\frac{1}{4}$ E) 1

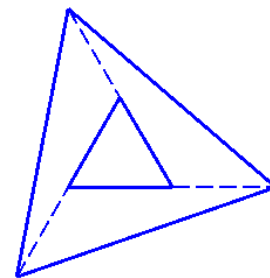
3. In a certain month, three Tuesdays fall on even-numbered dates. Which day of the week is the 28th day of the month?
- A) Friday B) Monday C) Sunday D) Tuesday
- E) There is not enough information to uniquely determine the day of the week.

4. Placed in numerical order (smallest to largest): $x = \frac{3}{3002002}$, $y = \frac{5}{5003009}$, $z = \frac{7}{7005001}$ is which of the 6 choices: x, y, z or x, z, y or y, x, z or y, z, x or z, x, y or z, y, x ?

5. In the following sum, each letter represents a unique digit. Thus, BB is a 2-digit number and CCC is a 3-digit number. What digit is represented by A ?

$$\begin{array}{r} A \\ A \\ + BB \\ \hline CCC \end{array}$$

6. Given an equilateral triangle with sides of length 1, a new equilateral triangle is formed by extending the three sides by 1 unit then connecting the new endpoints, as shown. What is the ratio of the area of the new triangle to the area of the original (smaller) one, in simplest form?



Problems 7 and 8 refer to the following cross-number puzzle. All answers are 3-digit numbers, so that none have a zero in the hundreds position.

Across

1. A permutation of the digits of a number that is both a perfect square and a perfect cube.
2. A perfect 4th power.
3. A perfect cube.

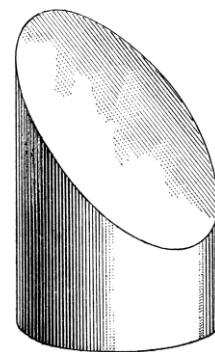
Down

1. A number.
2. An odd number whose reverse is a prime number.
3. A perfect 4th power.

1	2	3
2		
3		

7. What is "3 Across"?
8. What is "1 Down"?
9. An artist is planning on mixing together any number of different colors from her palette. A mixture results as long as the artist combines at least two colors. If the number of possible mixtures is less than 500, what is the greatest number of colors the artist could have in her palette?
10. A certain whole number between 2000 and 2100 has exactly 15 factors. What is the number?
For example: 10 has exactly 4 factors, namely 1, 2, 5, and 10.

11. A circular cylinder with a diameter of 1 has a plane cut made at an angle of 60° to the base, giving an elliptical cross-section, as shown. If the ellipse is then placed on the xy -plane with its center at the origin and major axis along the x -axis, then what is its equation?

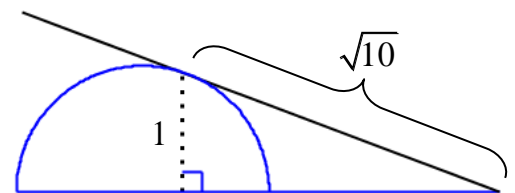


- A) $x^2 + 4y^2 = 1$ B) $x^2 + 3y^2 = 4$ C) $2x^2 + \sqrt{3}y^2 = 4$
D) $2x^2 + 3y^2 = 4$ E) $3x^2 + 4y^2 = 1$

12. What is the maximum value of the function $f(x) = \frac{2022}{\log(4x^2 - 4x + 101)}$, where $x \in (-\infty, \infty)$ and the logarithm is base 10?

- A) $\frac{2022}{\log(101)}$ B) 1011 C) 2022 D) 4044
E) There is no maximum value, the function is unbounded.

13. In triangle ABC , $\cos(3A - B) + \sin(A + 3B) = 2$. What is the *degree* measure of angle C ?
14. If $(x + 2)^2 + (y - 4)^2 = 45$ for real numbers x and y , then what is the maximum value of $x^2 + y^2$?
15. When the equation $\log_x(y) + 2022 = 2023\log_y(x)$ is solved for y in terms of x , two solutions are obtained. The product of the two solutions is x^n , where n is an integer. What is n ?
16. For the quadratic equation $x^2 + bx + r = 0$, the values for b and r are determined by the roll of dice. Two ordinary 6-sided dice (with the numbers 1 through 6), one blue and one red, are rolled. The number showing up on the blue die is assigned to b , and the number showing up on the red die is assigned to r . What is the probability the resulting quadratic equation will have at least one real solution?
- A) $\frac{1}{3}$ B) $\frac{5}{12}$ C) $\frac{1}{2}$ D) $\frac{19}{36}$ E) $\frac{7}{12}$
17. If 2022 is divided by 1, 2, 3, and so on up to 1000, then what is the largest remainder that can be obtained? For example, 2022 divided by 100 yields a remainder of 22, but that is not the largest possible remainder.
18. The graph of $y^3 - y^2 + y + x^2y - x^2 = 1$ is
- A) a point. B) a line. C) a parabola.
D) symmetric about the origin (i.e. an odd function). E) not a function.
19. The diagram shows a semicircle with a line resting on it (i.e. it is tangent to the semicircle). The dotted line represents the distance from the base of the semicircle to the point of tangency, and has length 1. The distance from the point of tangency to the point where the line intersects the extension of the base is $\sqrt{10}$, as shown. What is the length of the radius of the semicircle?



20. An inhabitant of Logic Land either speaks the truth all day or lies all day, but can never tell the truth and a lie on the same day. Which of the following statements can such a person *never* say?
- A) Yesterday, I told the truth. B) Yesterday, I lied. C) Today, I tell the truth.
D) Today, I lie. E) Tomorrow, I will lie.