Creating Effective Online Homework Problems in Algebra, Calculus, and Differential Equations

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WeBWorK

A rectangular room is 6 meters longer than it is wide, and its perimeter is 9 times the width. Find the dimensions of the room.

a. Write expressions for the length and perimeter in terms of w, where w is the width.

width: w

length: ______

perimeter: ______

a. Enter the equation that represents this situation.

Equation: ______

The length is ______ meters and the width is ______ meters.

The more we can do to **engage our students** more completely in the learning process, and to more rigorously reinforce the use of **correct notation**, **correct process**, **critical thinking** and the **synthesis of the course concepts**, the better these students should perform.

Online Homework problems in mathematics should ideally:

- Reinforce the **notation**, **concepts**, and **processes** learned in class.
- Challenge students to think carefully about concepts.
- Encourage students to ask questions when they don’t understand.

Many online HW problems only ask for a **single final answer** and this answer can often be found on the internet using tools such as Symbolab, PhotoMath, or Wolfram Alpha or even by just using their graphing calculator.

For example:

- Solving Word Problems!!
- Solving Rational Equations
- Limits
- Definite Integrals (even those requiring substitution)
- Improper Integrals
- Undetermined Coefficients
- Variation of Parameters

Is this the approach we want the students to take? Do we care about the process they use?

Students learn to do mathematics in the way they are assessed.

I believe that students are **trained** to work problems in the way they are led through them by the graded homework we assign.

If the online HW can be done most easily using the integrate feature on their calculator or by using Symbolab or Wolfram Alpha, etc. and getting the final answer, this is how we are training them to do these problems. This is what they are practicing.

Have you ever had a student tell you:

"I got the right answer, but could you still explain how to do this problem?"

I have. On a definite integral problem that required only a numerical approximation of the answer.

(1 pt) **local/Library/Union/setIntFTC/an6_6_4.pg**
Use the Fundamental Theorem of Calculus to evaluate the definite integral.

$$\int_{-1}^{1} \frac{3}{x^2+1}\, dx = \underline{\qquad\qquad} \Big|_{-1}^{1} = \underline{\qquad\qquad}$$

Hint: Be sure to give your final answer in exact form, and not as a decimal. Remember that you can enter π by typing **pi** .

Some Best Practices for Creating Online Homework Problems for Mathematics:

1. Ask for **intermediate results** whenever appropriate (and helpful), especially for long problems or for problems where the answer could easily be found using a calculator or online tool.

- To **train students to show the steps we would require in their written work on exams.** We want to train students to think through the entire problem instead of simply trying to obtain a correct final answer.

- To **reduce student frustration** on long complicated problems, **let them know if they are correct for each step** as they go through the problem, e.g.,
  - Trig. Substitution Integrals,
  - Volumes of a Common Cross-Section,
  - Optimization Problems, etc.

They can tell if they are on the right track early in the problem.

By asking for meaningful steps, we help the student focus on the process rather than only on the final answer.

Ken drove a total of 276.5 miles on a two-part road trip. In the first part of the trip, he drove for 3.5 hours. In the second part of the trip, he drove an average of 9 miles per hour faster, and drove for 2 hours. What was Ken's average speed over each leg of the trip?

You can use a table to organize the quantities (speed, time, and distance) for each leg of Ken's trip. If we let x represent Ken's average speed for the first leg (in miles per hour), then his average speed over the second leg is 9 miles per hour faster. Write an expression for this in the table below.

Organize data with a table

	Rate	$\times$	**Time**	$=$	**Distance**
first leg of trip	x		3.5		3.5x
second leg of trip	x + 9		2		2(x + 9)

According to the table, the total distance that Ken traveled is 3.5x + 2(x + 9) miles (in terms of x).

Now enter an equation in terms of x that we can solve to find Ken's average speeds.
3.5x + 2(x + 9) = 276.5

Ken had an average speed of 47 miles per hour over the first leg of the trip.

Ken had an average speed of 56 miles per hour over the second leg of the trip.

A rectangular room is 6 meters longer than it is wide, and its perimeter is 9 times the width. Find the dimensions of the room.

a. Write expressions for the length and perimeter in terms of w, where w is the width.

width: w

length:

perimeter:

a. Enter the equation that represents this situation.

Equation:

The length is ______ meters and the width is ______ meters.

Tammy and Ron both purchase apples and plums at the market. Tammy buys 4 pounds of apples and 8 pounds of plums for $46. Ron buys 9 pounds of apples and 15 pounds of plums for $90.75.

Select the correct definition for each of the variables in the following setup. Then enter a system of two equations to represent this situation and use them to solve the problem.

Price per pound of apples : a

Price per pound of plums : p

Equation 1: 4a + 8p = 46

Equation 2: 9a + 15p = 90.75

Apples cost $ 3 per pound and plums cost $ 4.25 per pound.

(1 pt) **local/Library/ma123DB/set3/s7_4_43Seeburger.pg**
Use the Fundamental Theorem of Calculus and the Substitution Method to find

$$\int_0^1 x^2\sqrt{6x+9}\,dx$$

First, using u to represent the inside function, what would it be defined to be?

$u =$ ____ so $x =$ ____

$du =$ ____ so $dx =$ ____

Fill in the integrand below to rewrite the original integral in terms of the new variable u, indicating the new limits of integration as values of u.

$$\int_0^1 x^2\sqrt{6x+9}\,dx$$

$$= \int_{\square}^{\square} \text{____}$$

Now finish by giving the final value of the definite integral.

$$\int_0^1 x^2\sqrt{6x+9}\,dx = \text{____}$$

(2 pts) **local/Library/272/setStewart15_3/problem_4a.pg**
Evaluate the integral by reversing the order of integration.

$$\int_0^3 \int_{y^2}^9 y\cos(x^2)\, dx dy =$$

$$= \int_{\square}^{\square} \int_{\square}^{\square} \square \, dy\, dx$$

= $\square$

Be sure to enter your final answer in **exact** form.

(2 pts) **local/Problems/CalcIII/Double_Integrals/UR_VC_9_1.pg**
Using polar coordinates, evaluate the integral

$\iint_R \sin(x^2+y^2)dA$, where R is the region $9 \le x^2+y^2 \le 64$.

$$\iint_R \sin(x^2+y^2)dA$$

$$= \int_{\square}^{\square} \int_{\square}^{\square} \square \, dr\, d\theta$$

= $\square$

Be sure to enter your final answer in **exact** form.

(4 pts) **local/Seeburger/Ch13-4Differential1.pg**
Find the total differential of the function $z = y^3x^4 - 10y - 13$.
$dz =$ [ ]

Now calculate Δz and dz for $x = 1, y = -2$, and $f(0.97, -2.08)$.

Enter Δz and dz without rounding off.
Also show the work requested below.

$\Delta z = f($ [ ] $) - f($ [ ] $) \approx$ [ ]

$dx =$ [ ] and $dy =$ [ ] , so

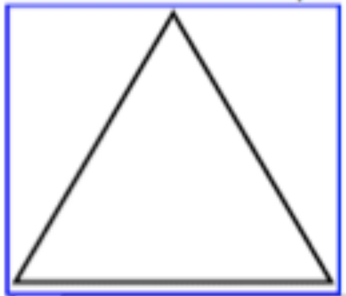
$dz = f_x($ [ ] $)dx + f_y($ [ ] $)dy =$ [ ]

SOLUTION:

(1 pt) **local/Library/Michigan/Chap8Sec2/Q27.pg**

Find the volume of the solid whose base is the region bounded by $y = x^5$, $y = 1$, and the y-axis and whose cross-sections perpendicular to the **y-axis** are equilateral triangles.

The base of this equilateral triangle, in terms of the variable y is:



b = ______ .

The height of this equilateral triangle, in terms of the variable y is:

h = ______ .

The area of this equilateral triangle, in terms of the variable y is:

$A(y)$ = ______ .

Fill in the integral below to represent the Total Volume of this solid.

$$\text{Total Volume} = \int_{\square}^{\square} \square$$

Now give the exact volume.

Total Volume = ______ units^3.

SOLUTION:

(5 pts) For each nonhomogeneous differential below, write the simplest possible form for the particular solution of the DE (that is, show the least complicated form that will work). Use capital letters A, B, C, etc. for the undetermined coefficients, starting with A.

To help you, the general solutions of the corresponding homogeneous DEs are given. In these given solutions c_1 and c_2 represent arbitrary constants.

a. Differential equation: $y'' - 12y' + 35y = 9t + 4e^{5t}$

   $y_c = c_1 e^{5t} + c_2 e^{7t}$

   Form of $y_p =$ At+B+Cte^(5t) help (formulas)

b. Differential equation: $y'' - 5y' = 8t^2 + 4t - 4e^{-5t}$

   $y_c = c_1 + c_2 e^{5t}$

   Form of $y_p =$ At^3+Bt^2+Ct+De^(-5t) help (formulas)

c. Differential equation: $y'' - 5y' = 4t^2 + 4e^{5t}$

   $y_c = c_1 + c_2 e^{5t}$

   Form of $y_p =$ At^3+Bt^2+Ct+Dte^(5t) help (formulas)

d. Differential equation: $y'' - 10y' + 41y = 8\cos(4t)$

   $y_c = c_1 e^{5t}\cos 4t + c_2 e^{5t}\sin 4t$

   Form of $y_p =$ Acos(4t)+Bsin(4t) help (formulas)

e. Differential equation: $y'' - 10y' + 41y = 8e^{5t}\sin(4t) + 4e^{5t}$

   $y_c = c_1 e^{5t}\cos 4t + c_2 e^{5t}\sin 4t$

   Form of $y_p =$ Ae^(5t)+e^(5t)(Btcos(4t)+Ctsin(4t)) help (formulas)

(8 pts) Use variation of parameters to find a particular solution to: $y'' - 6y' + 9y = \frac{14.5e^{3t}}{t^2+1}$.

a. Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants. Enter c_1 as `c1` and c_2 as `c2`.

$y_c =$ c1e^(3t)+c2te^(3t)

b. Use the two independent solutions of the homogeneous form of the DE to enter the Wronskian determinant, W:

$W = \det \begin{bmatrix} \text{e^(3t)} & \text{te^(3t)} \\ \text{3e^(3t)} & \text{3te^(3t)+e^(3t)} \end{bmatrix} =$ e^(6t)

c. Now enter the determinants, W_1 and W_2:

$W_1 = \det \begin{bmatrix} \text{0} & \text{te^(3t)} \\ \text{(14.5e^(3t))/(t^2+1)} & \text{3te^(3t)+e^(3t)} \end{bmatrix} =$ -(14.5te^(6t))/(t^2+1)

$W_2 = \det \begin{bmatrix} \text{e^(3t)} & \text{0} \\ \text{3e^(3t)} & \text{(14.5e^(3t))/(t^2+1)} \end{bmatrix} =$ (14.5e^(6t))/(t^2+1)

d. Use these results to determine u_1' and u_2'. (Please simplify them. This will make it easier to integrate in the next step.)

$u_1' = \frac{W_1}{W} =$ -(14.5t)/(t^2+1)

$u_2' = \frac{W_2}{W} =$ (14.5)/(t^2+1)

e. Now integrate u_1' to determine u_1,

$u_1 = \int u_1'\, dt =$ -(14.5/2)ln|t^2+1| $+ \; C_1$

and integrate u_2' to determine u_2.

$u_2 = \int u_2'\, dt =$ 14.5arctan(t) $+ \; C_2$

f. Finally, enter the simplest form of the particular solution you obtained (not including any terms that are part of the complementary solution to the homogeneous form of this DE).

$y_p =$ 14.5(te^(3t))arctan(t)-(14.5(e^(3t))/2)ln|t^2+1|

2. [Include a clear worked-out solution] for each problem.

Students can then understand their mistakes and get a clearer understanding of the problem procedures and concepts as they review for exams.

SOLUTION:

SOLUTION: (Instructor solution preview: show the student solution after due date.)

SOLUTION

The base of the equilateral triangle can be found by using a horizontal rectangle (perpendicular to the y -axis) and subtracting right curve - left curve. This yields: $b = \sqrt[5]{y} - 0 = \sqrt[5]{y}$.

An equilateral triangle of side s has height $\frac{\sqrt{3}s}{2}$. Here $h = \frac{\sqrt{3}}{2} \cdot \sqrt[5]{y}$.

So the area of this equilateral triangle is $\frac{1}{2} b \cdot h = \frac{1}{2} \sqrt[5]{y} \cdot \frac{\sqrt{3}}{2} \cdot \sqrt[5]{y} = \frac{\sqrt{3}}{4} (\sqrt[5]{y})^2$.
[You can use one of the two right triangles you get by dividing this equilateral triangle in half and the Pythagorean Theorem to verify this.]

Thus

$$\text{Area of a triangular slice } A(y) = \frac{\sqrt{3}}{4} (\sqrt[5]{y})^2 = \frac{\sqrt{3}}{4} y^{2/5}.$$

So, since the thickness of the triangular cross-section is Δy,

$$\text{Volume of a triangular slice } V(y) = \frac{\sqrt{3}}{4} (\sqrt[5]{y})^2 \Delta y = \frac{\sqrt{3}}{4} y^{2/5} \Delta y.$$

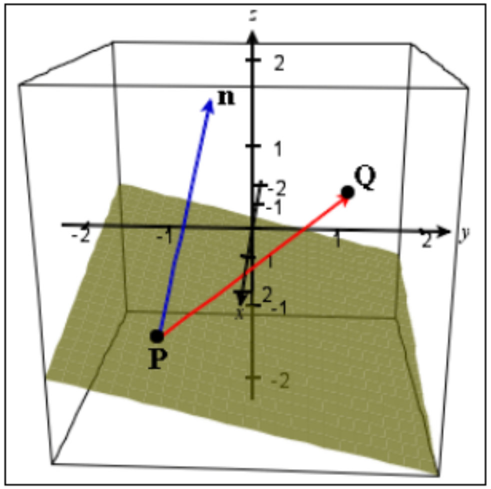
Therefore,

$$\text{Total Volume of the Solid } = \int_0^1 A(y)\, dy = \int_0^1 \frac{\sqrt{3}}{4} y^{2/5}\, dy = \frac{\sqrt{3}}{4} \frac{y^{2/5+1}}{(7/5)} \Big|_0^1 = \frac{5\sqrt{3}}{28} \text{ units}^3.$$

3. Be sure to **use correct notation** and format the objects in the problem in a way that you would want them to write them for your exams.

(1 pt) **local/Seeburger/ch11_5/pointPlaneDist.pg**

To find the distance d of a point Q to the plane containing a point P with normal vector $\vec{\mathbf{n}}$ (shown in the diagram), we will need a way to find the shortest distance from the point Q to the plane.
Which of the following formulas will correctly give the **distance** d of the point Q from this plane?



- ○ **A.** $\|\overrightarrow{\mathbf{PQ}}\|$
- ○ **B.** $\|\vec{\mathbf{n}}\|$
- ○ **C.** $\|\mathrm{Proj}_{\overrightarrow{\mathbf{PQ}}}\vec{\mathbf{n}}\|$
- ○ **D.** $\|\mathrm{Proj}_{\vec{\mathbf{n}}}\overrightarrow{\mathbf{PQ}}\|$
- ○ **E.** $\|\overrightarrow{\mathbf{PQ}} - \vec{\mathbf{n}}\|$
- ○ **F.** $\|\vec{\mathbf{n}} - \mathrm{Proj}_{\vec{\mathbf{n}}}\overrightarrow{\mathbf{PQ}}\|$
- ○ **G.** $\|\vec{\mathbf{n}} \times \overrightarrow{\mathbf{PQ}}\|$
- ○ **H.** $\|\overrightarrow{\mathbf{PQ}} - \mathrm{Proj}_{\vec{\mathbf{n}}}\overrightarrow{\mathbf{PQ}}\|$

SOLUTION:

(2 pts) **local/Seeburger/ch13_1/domain3.pg**

State the **Domain** of f, i.e., the largest set on which the function $f(x,y) = \dfrac{17}{\sqrt{9 - x^2 - y^2}}$ is continuous.

Insert the correct condition in the set-builder notation below.
Be sure to use the clearest possible form for the condition.

D_f : $\{(x,y)$: x^2 + y^2 < 9 $\}$

Now enter the **Range** of f using interval notation.
(Note: You can enter ∞ as $\inf$ and $-\infty$ as $\inf$.)

R_f : [17/sqrt(9), inf)

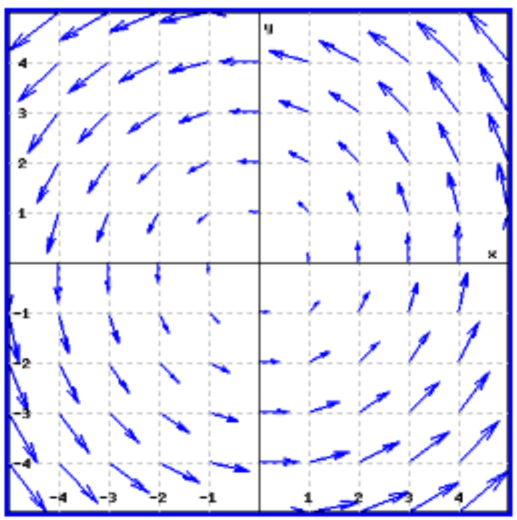
4. Take advantage of **visualization** whenever possible and ask **conceptual questions** whenever possible.

(2 pts) **Library/FortLewis/Calc3/17-3-Vector-fields/HGM5-17-3-27-Vector-fields.pg**
Match each vector field with its graph.

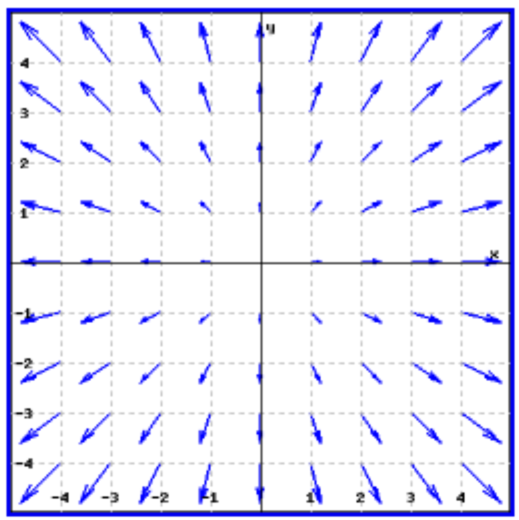
? **1.** $\vec{H}(x,y) = x\vec{i} - y\vec{j}$

? **2.** $\vec{G}(x,y) = -y\vec{i} + x\vec{j}$

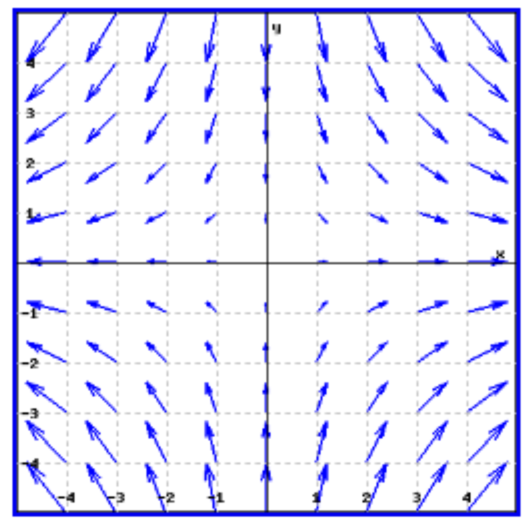
? **3.** $\vec{F}(x,y) = x\vec{i} + y\vec{j}$



A



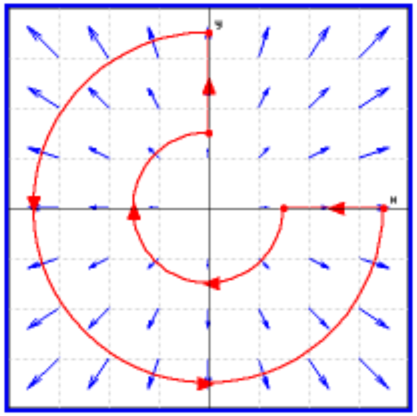
B



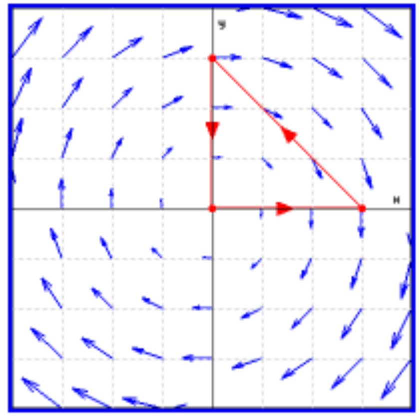
C

(Click on a graph to enlarge it)

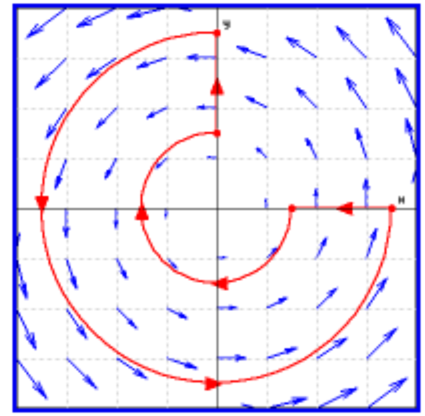
(1 pt) **Library/FortLewis/Calc3/18-1-Idea-of-line-integrals/HGM4-18-1-02-Line-integrals.pg**
Determine whether the line integral of each vector field (in blue) along the oriented path (in red) is positive, negative, or zero.



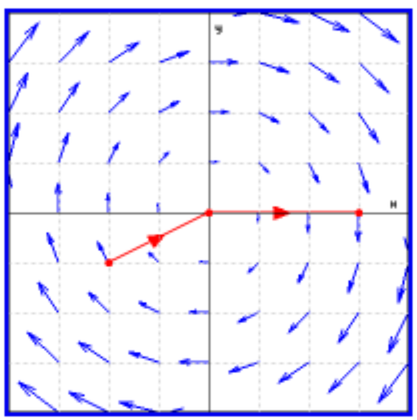
Zero



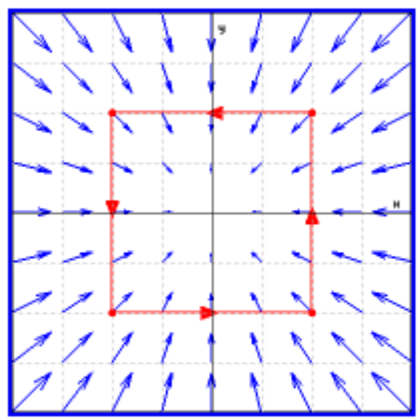
Negative



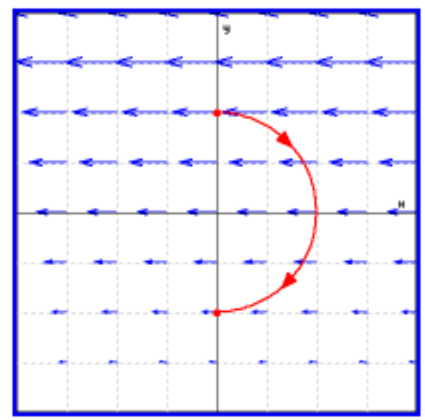
Positive



Zero



Zero



Negative

5. Where appropriate **include helpful hints**, intelligent **feedback** ("This answer should include units." or "This answer should be an ordered triple.") and links to help pages or online resources relevant to the problem.

   This support will encourage students to keep working at it.

(1 pt) **local/Library/UMN/calculusStewartCCC/s_2_5_20.pg**
Find the following limit, showing the last step in the analytical process of determining the limit, just before you were able to evaluate the limit.
If the limit goes to ∞, write "inf". If a limit goes to $-\infty$, write "-inf".

$$\lim_{x\to 3} \frac{x^2-3x}{x^2-2x-3} = \quad \cdots \quad = \lim_{x\to 3} \frac{\square}{\square}$$

Limit: $\square$

HINT:

HINT: (Instructor hint preview: show the student hint after 3 attempts. The current number of attempts is 0.)
Factor and cancel the common factor to remove the discontinuity.

6. When appropriate, **display** these multistep problems **one step at a time**, instead of overwhelming them with all the steps at once.

(1 pt) **local/Seeburger/IntAlg/RationalEq/Problem1.pg**

Solve the equation $\frac{6}{x+1} - \frac{1}{2} = \frac{4}{3x+3}$.

Step 1 (of 3): Factor the denominators and state the Least Common Deonominator (LCD) of the fractions in the equation.

LCD = 6(x+1)

Step 2 (of 3): Multiply both sides of the equation by this LCD and simplify each side to cancel the denominators.

$$6(x+1)\cdot\left(\frac{6}{x+1} - \frac{1}{2}\right) = 6(x+1)\cdot\frac{4}{3(x+1)}$$

Enter the resulting expressions you obtain on each side of the equation.

36 - 3(x+1) = 8

Step 3 (of 3): Now finish solving the equation for x.

$x =$ 28/3 - 1

7. Whenever appropriate, be sure to ask students for **appropriate units**.

- **Be sure the instructions are clear and specific.** If you expect students to give answers in a particular form, be sure to state this. For example, if you require the answer to be given in exact form using π instead of as a decimal approximation, tell them this.

- **Be sure the problem works!** Check that the result can be consistently checked (i.e., it must be numerically stable).

- Be sure the randomized values in a problem do not (even sometimes) make the calculations and the answer more complicated than you intend.

- Be careful that the correct answer that displays once the homework set is due is displayed in exact form, if this is what you requested from the student in the problem.

- Include problems that connect the course concepts to **real-world contexts** whenever possible. This relevance provides intrinsic motivation for our students and improves student outcomes.

Thank you!

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