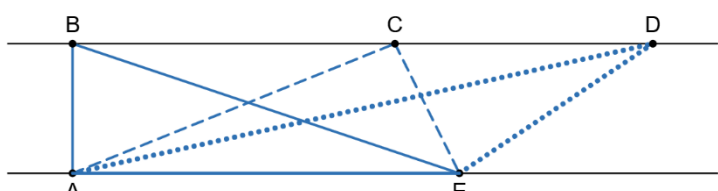


New York State Mathematics Association of Two-Year Colleges

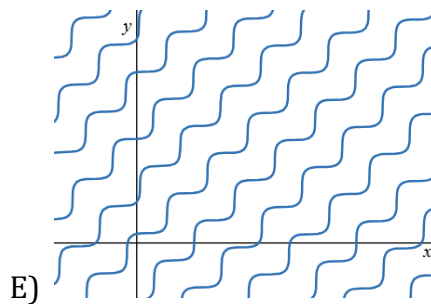
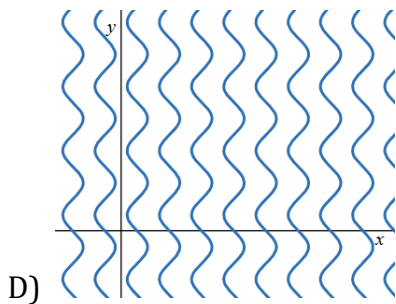
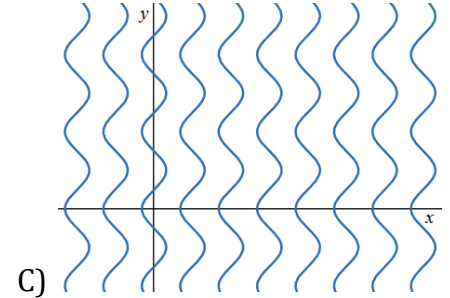
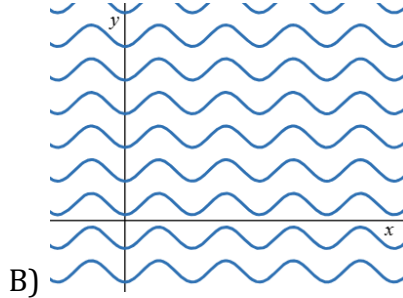
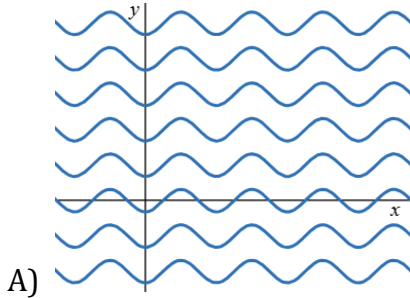
Math League Contest ~ Fall 2025

Directions: You have one hour to take this test. Scrap paper is allowed. The use of calculators is NOT permitted, as well as computers, books, math tables, and notes of any kind. You are not expected to answer all the questions. However, do not spend too much time on any one problem. Four points are awarded for each correct answer, one point is deducted for each incorrect answer, and no points are awarded/deducted for blank responses. There is no partial credit. Unless otherwise indicated, answers must be given in *exact* form, i.e. in terms of fractions, radicals, π , etc.

- What is the domain of the function $f(x) = \sqrt{1 - \sqrt{2 - \sqrt{3 - x}}}$?
- The numbers: 2^{500} , 5^{200} , and 9^{150} placed in numerical order, from smallest to largest, are?
 A) 2^{500} , 5^{200} , 9^{150} B) 2^{500} , 9^{150} , 5^{200} C) 5^{200} , 2^{500} , 9^{150}
 D) 5^{200} , 9^{150} , 2^{500} E) 9^{150} , 5^{200} , 2^{500}
- The year 2025 is a perfect square (which will not happen again for 91 years!). How many positive integers are factors of 2025^{25} ?
- $\log_8(27) \cdot \log_9(125) \cdot \log_{25}(32) = ?$
 A) $\log_{8 \cdot 9 \cdot 25}(27 \cdot 32 \cdot 125)$ B) $\log_{42}(27 \cdot 32 \cdot 125)$ C) $\frac{27}{8}$ D) $\frac{15}{4}$ E) $\frac{16}{3}$
- Three triangles, $\triangle ABE$ – the only right triangle, $\triangle ACE$, and $\triangle ADE$, all with common base AE , are drawn between two parallel lines, as shown. Which triangle has the greatest area?
 A) $\triangle ABE$ B) $\triangle ACE$ C) $\triangle ADE$ D) They all have the same area.
 E) There is not enough information to make a determination.
 
- A Pythagorean Triple is a set of three positive integers a , b , and c , such that $a^2 + b^2 = c^2$ (i.e. line segments with those lengths form a right triangle, e.g. 3, 4, and 5). How many such triples are there where each integer is an odd number?
 A) 0 B) 1 C) 2 D) 3 E) infinitely many

7. There are two values for k that make the system of equations: $3x + ky = 4$ and $kx + 6y = 1$ have no solution, call them k_1 and k_2 . What is $k_1 \cdot k_2$?
- A) -18 B) -9 C) 0 D) 9 E) 18

8. Which of the following graphs most closely depicts the graph of $\sin(x + \cos(y)) = 0$?



9. The function $f(x) = \frac{x}{2^x - 1} + \frac{x}{2}$ is

- A) an even function, but not odd. B) an odd function, but not even.
 C) an even and odd function. D) neither an even nor odd function.
 E) not a function.

10. What is the shortest distance between the graph of $y^2 + 2y - 10 = 6x - x^2$ and the point $(2, 1)$?

- A) $\sqrt{10} - 1$ B) $\sqrt{5}$ C) $\sqrt{11} - 1$ D) $\sqrt{19} - 2$ E) $\sqrt{6}$

11. The equation $\frac{2}{|x+1|} - \frac{1}{|x+2|} = 1$ has two real solutions, call them x_1 and x_2 with $x_1 < x_2$. What is the value of $x_2 - x_1$?

12. If $\log_b(xy^3) = 9$ and $\log_b(x^2y) = 13$, then what is $\log_b(xy)$?

- A) 5 B) 6 C) 7 D) 8
 E) It cannot be determined without knowing the value of b (i.e. the base).

13. The prime factorization of $100!$ will have 2^n , among many other prime factors. What is the value of n ?
 In other words, what is the largest integer power of 2 for which 2^n is a factor of $100!$?
 Note: The Factorial function is defined as $n! = n(n-1)(n-2) \dots 2 \cdot 1$, e.g. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

14. $\sqrt{7+4\sqrt{3}} - \sqrt{7-4\sqrt{3}} = ?$
 A) 3 B) $\sqrt{11}$ C) $2\sqrt{3}$ D) $3\sqrt[4]{2}$ E) $1+2\sqrt[4]{3}$

15. What is the value of the sum $\sum_{x=1}^{89} \ln[\tan(x^\circ)]$?

Note: $\sum_{x=1}^{89} \ln[\tan(x^\circ)] = \ln[\tan(1^\circ)] + \ln[\tan(2^\circ)] + \ln[\tan(3^\circ)] + \dots + \ln[\tan(89^\circ)]$

A) -1 B) 0 C) $\frac{1}{2}$ D) $\ln(2)$ E) 1

16. The equation $2^x = 3x$ has two solutions. The sum of those solutions is in which interval?
 A) (1,2) B) (2,3) C) (3,4) D) (4,5) E) (5,6)

17. Sophia summed the page numbers of a journal she was reading and obtained 2025. However, she accidentally counted a particular page number twice. Which page number was counted twice?
 Assume the journal pages are numbered from 1 to n , where n is the last page number.

18. The following addition problem is called an *alphametic puzzle*, where the objective is to determine what digit each letter represents in order to make the equation true.

$$\text{ONE} + \text{ONE} + \text{ONE} + \text{ONE} + \text{ONE} + \text{ONE} + \text{ONE} + \text{ONE} + \text{ONE} = \text{NINE}$$

This one has two different solutions. One of the solutions has E representing zero. For this solution, what digit does the I (in NINE) represent? Note: Each letter represents a unique digit, i.e. no digit can be represented by two different letters.

19. The equation $x = 1 - 6(1 - 6x^2)^2$ has exactly two rational roots. What is the sum of those two roots?

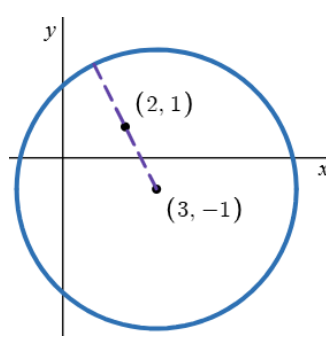
20. One hundred people line up to board an airplane with 100 passenger seats. Each has a boarding pass with their assigned seat. However, the first person to board has lost his boarding pass and takes a random seat. After that, each person takes their assigned seat if it is unoccupied, and one of the unoccupied seats at random otherwise. What is the probability that the last person to board gets to sit in his/her assigned seat?



New York State Mathematics Association of Two-Year Colleges

Math League Contest ~ Fall 2025 ~ Solutions

1.	We need: ① $3 - x \geq 0 \Rightarrow x \leq 3$, and ② $\sqrt{3 - x} \leq 2 \Rightarrow x \geq -1$, and ③ $\sqrt{2 - \sqrt{3 - x}} \leq 1 \Rightarrow \sqrt{3 - x} \geq 1 \Rightarrow x \leq 2$. Thus, in order to satisfy all three inequalities, $-1 \leq x \leq 2$ or $x \in [-1, 2]$. Answer: $-1 \leq x \leq 2$ or $x \in [-1, 2]$
2.	$2^{500} = 2^{5 \cdot 100} = 32^{100}$, $5^{200} = 5^{2 \cdot 100} = 25^{100}$, $9^{150} = 9^{1.5 \cdot 100} = 9^{\frac{3}{2} \cdot 100} = 27^{100}$ Now it is clear to see that $25^{100} < 27^{100} < 32^{100} \Rightarrow 5^{200} < 9^{150} < 2^{500}$. Answer: D
3.	$2025 = 45^2 = (9 \cdot 5)^2 = (3^2 \cdot 5)^2 = 3^4 \cdot 5^2$. Thus, $2025^{25} = (3^4 \cdot 5^2)^{25} = 3^{100} \cdot 5^{50}$. Factors must be of the form $3^m \cdot 5^n$, where m is an integer from 0 through 100 and n is an integer from 0 through 50. Since m can take on any one of 101 values and n can take on 51 values, there are $101 \cdot 51 = 5151$ different factors. Answer: 5151
4.	$\log_8(27) \cdot \log_9(125) \cdot \log_{25}(32) = \frac{\log(27)}{\log(8)} \cdot \frac{\log(125)}{\log(9)} \cdot \frac{\log(32)}{\log(25)} = \frac{\log(32)}{\log(8)} \cdot \frac{\log(27)}{\log(9)} \cdot \frac{\log(125)}{\log(25)}$ $= \frac{\log_2(32)}{\log_2(8)} \cdot \frac{\log_3(27)}{\log_3(9)} \cdot \frac{\log_5(125)}{\log_5(25)} = \frac{5}{3} \cdot \frac{3}{2} \cdot \frac{3}{2} = \frac{15}{4}$ Answer: D
5.	All three triangles have the same base, and the same height – since height is the perpendicular distance from the base to the opposite vertex. Since the area of a triangle is $\frac{1}{2}(\text{base})(\text{height})$, they all have the same area. Answer: D
6.	Since the square of an odd number is odd $a^2 + b^2$ would be the sum of two odd numbers, which is even. Making it impossible for c to be odd. Thus, there are no such Pythagorean Triples. Answer: A
7.	Putting each linear function in slope-intercept form yields: $y = -\frac{3}{k}x + \frac{4}{k}$ and $y = -\frac{k}{6}x + \frac{1}{k}$. In order for there to be no solution, the two lines must not intersect. Thus, they must have equal slopes with different y-intercepts. Setting the two slopes equal and solving for k: $-\frac{3}{k} = -\frac{k}{6} \Rightarrow k^2 = 18 \Rightarrow k = \pm\sqrt{18}$. Hence, the product of the two values for k is -18. Answer: A
8.	Since the sine of any integer multiple of π is zero (i.e. $\sin(n\pi) = 0$, where n is an integer), we can write the equation as $x + \cos(y) = n\pi$ or $x = n\pi - \cos(y)$, for any integer n. The graph of these is a family of cosine curves where each one has x as a function of y (i.e. the cosine functions run vertically). Thus, choices A, B, and E are eliminated. When $n = 0$, we get $x = -\cos(y)$, which eliminates choice D, since no cosine curve intersects the y-axis. Answer: C

9.	We need to check if $f(-x) = f(x)$, f is even; or if $f(-x) = -f(x)$, f is odd -- or neither. $f(-x) = \frac{-x}{2^{-x}-1} + \frac{-x}{2} = \frac{-x}{2^{-x}-1} \cdot \frac{2^x}{2^x} - \frac{x}{2} = \frac{-x \cdot 2^x}{1-2^x} - \frac{x}{2} = \frac{-x \cdot 2^x + x - x}{1-2^x} - \frac{x}{2} = \frac{(x - x \cdot 2^x) - x}{1-2^x} - \frac{x}{2}$ $= \frac{x(1-2^x) - x}{1-2^x} - \frac{x}{2} = x - \frac{x}{1-2^x} - \frac{x}{2} = \frac{x}{2^x-1} + x - \frac{x}{2} = \frac{x}{2^x-1} + \frac{x}{2} = f(x)$ Hence, $f(x)$ is an even function. Answer: A
10.	Completing the square for both x and y variables will help identify the graph. $x^2 - 6x + (-3)^2 + y^2 + 2y + 1^2 = 10 + (-3)^2 + 1^2 \Rightarrow (x-3)^2 + (y+1)^2 = 20$ Thus, it is a circle centered at $(3, -1)$ with a radius of $\sqrt{20} = 2\sqrt{5}$, as shown. Hence, the point $(2, 1)$ lies inside the circle. The shortest distance will be along the radius through the point $(2, 1)$. The distance between $(2, 1)$ and the center of the circle $(3, -1)$ is $\sqrt{(3-2)^2 + (-1-1)^2} = \sqrt{5}$. Therefore, the shortest distance is $2\sqrt{5} - \sqrt{5} = \sqrt{5}$. Answer: B 
11.	Let's consider all cases for the two absolute value functions. Case 1: $x < -2 \Rightarrow \frac{2}{-(x+1)} - \frac{1}{-(x+2)} = 1 \Rightarrow \frac{-2}{x+1} + \frac{1}{x+2} = 1 \Rightarrow -2(x+2) + x+1 = (x+1)(x+2)$ $\Rightarrow -x-3 = x^2+3x+2 \Rightarrow x^2+4x+5=0 \Rightarrow \text{non-real solutions}$ Case 2: $-2 < x < -1 \Rightarrow \frac{2}{-(x+1)} - \frac{1}{x+2} = 1 \Rightarrow \frac{-2}{x+1} - \frac{1}{x+2} = 1 \Rightarrow -2(x+2) - (x+1) = (x+1)(x+2)$ $\Rightarrow -3x-5 = x^2+3x+2 \Rightarrow x^2+6x+7=0 \Rightarrow x = -3 \pm \sqrt{2}, \text{ but } -3 - \sqrt{2} < -2$ So only $x = -3 + \sqrt{2}$ in this case. Case 3: $x > -1 \Rightarrow \frac{2}{x+1} - \frac{1}{x+2} = 1 \Rightarrow 2(x+2) - (x+1) = (x+1)(x+2) \Rightarrow x+3 = x^2+3x+2$ $\Rightarrow x^2+2x-1=0 \Rightarrow x = -1 \pm \sqrt{2}, \text{ but } -1 - \sqrt{2} < -1 \text{ So only } x = -1 + \sqrt{2} \text{ in this case.}$ This gives $x_2 = -1 + \sqrt{2}$ and $x_1 = -3 + \sqrt{2}$. Hence, $x_2 - x_1 = 2$. Answer: 2
12.	$\log_b(xy^3) = 9 \Rightarrow \textcircled{1} \log_b(x) + 3\log_b(y) = 9$ and $\log_b(x^2y) = 13 \Rightarrow \textcircled{2} 2\log_b(x) + \log_b(y) = 13$ Solving equations $\textcircled{1}$ and $\textcircled{2}$ for $\log_b(x)$ and $\log_b(y)$ gives: $\log_b(x) = 6$ and $\log_b(y) = 1$. This gives: $\log_b(x) + \log_b(y) = 6 + 1 \Rightarrow \log_b(xy) = 7$. Answer: C
13.	$100! = 100 \cdot 99 \cdot 98 \cdot \dots \cdot 3 \cdot 2 \cdot 1$ Each multiple of 2 contributes a factor of 2, which gives $100 \div 2 = 50$. Each multiple of 4 contributes another factor of 2, which gives $100 \div 4 = 25$. Each multiple of 8 contributes another one, which gives $100 \div 8 = 12.5$ which rounds down to 12. Each multiple of 16 contributes another, which gives $100 \div 16 = 6.25$ which rounds down to 6. Each multiple of 32 contributes another, which gives $100 \div 32$ which rounds down to 3. Each multiple of 64 contributes one more, which gives $100 \div 64$ which rounds down to 1. Thus, there are $50 + 25 + 12 + 6 + 3 + 1 = 97$ 2's that factor out of $100!$. Answer: 97

14.	Let $x = \sqrt{7+4\sqrt{3}} - \sqrt{7-4\sqrt{3}} \Rightarrow x^2 = 7+4\sqrt{3} - 2\sqrt{7+4\sqrt{3}} \cdot \sqrt{7-4\sqrt{3}} + 7-4\sqrt{3}$ $\Rightarrow x^2 = 14 - 2\sqrt{(7+4\sqrt{3}) \cdot (7-4\sqrt{3})} = 14 - 2\sqrt{49-48} = 12 \Rightarrow x = \pm\sqrt{12}$ However, $x > 0$ so that $x = \sqrt{12} = 2\sqrt{3}$. Answer: C
15.	$\ln[\tan(1^\circ)] + \ln[\tan(2^\circ)] + \ln[\tan(3^\circ)] + \dots + \ln[\tan(89^\circ)] = \ln[\tan(1^\circ) \cdot \tan(2^\circ) \cdot \tan(3^\circ) \cdot \dots \cdot \tan(89^\circ)]$ $= \ln\left[\frac{\sin(1^\circ)}{\cos(1^\circ)} \cdot \frac{\sin(2^\circ)}{\cos(2^\circ)} \cdot \frac{\sin(3^\circ)}{\cos(3^\circ)} \cdot \dots \cdot \frac{\sin(44^\circ)}{\cos(44^\circ)} \cdot \frac{\sin(45^\circ)}{\cos(45^\circ)} \cdot \frac{\sin(46^\circ)}{\cos(46^\circ)} \cdot \dots \cdot \frac{\sin(87^\circ)}{\cos(87^\circ)} \cdot \frac{\sin(88^\circ)}{\cos(88^\circ)} \cdot \frac{\sin(89^\circ)}{\cos(89^\circ)}\right]$ However, since $\sin(x^\circ) = \cos(90^\circ - x^\circ)$ and $\cos(x^\circ) = \sin(90^\circ - x^\circ)$ the expression above can be written as: $\ln\left[\frac{\sin(1^\circ)}{\cos(1^\circ)} \cdot \frac{\sin(2^\circ)}{\cos(2^\circ)} \cdot \frac{\sin(3^\circ)}{\cos(3^\circ)} \cdot \dots \cdot \frac{\sin(44^\circ)}{\cos(44^\circ)} \cdot \frac{\sin(45^\circ)}{\cos(45^\circ)} \cdot \frac{\cos(44^\circ)}{\sin(44^\circ)} \cdot \dots \cdot \frac{\cos(3^\circ)}{\sin(3^\circ)} \cdot \frac{\cos(2^\circ)}{\sin(2^\circ)} \cdot \frac{\cos(1^\circ)}{\sin(1^\circ)}\right]$ $= \ln\left[\frac{\cos(1^\circ)}{\cos(1^\circ)} \cdot \frac{\cos(2^\circ)}{\cos(2^\circ)} \cdot \frac{\cos(3^\circ)}{\cos(3^\circ)} \cdot \dots \cdot \frac{\cos(44^\circ)}{\cos(44^\circ)} \cdot \frac{\sin(45^\circ)}{\cos(45^\circ)} \cdot \frac{\sin(44^\circ)}{\sin(44^\circ)} \cdot \dots \cdot \frac{\sin(3^\circ)}{\sin(3^\circ)} \cdot \frac{\sin(2^\circ)}{\sin(2^\circ)} \cdot \frac{\sin(1^\circ)}{\sin(1^\circ)}\right]$ $= \ln\left[1 \cdot 1 \cdot 1 \cdot \dots \cdot 1 \cdot \frac{\sin(45^\circ)}{\cos(45^\circ)} \cdot 1 \cdot \dots \cdot 1 \cdot 1 \cdot 1\right] = \ln\left[\frac{\sin(45^\circ)}{\cos(45^\circ)}\right] = \ln[\tan(45^\circ)] = \ln[1] = 0$ Answer: B Alternate Solution: Since $\tan(x^\circ) = \cot(90^\circ - x^\circ) = 1/\tan(90^\circ - x^\circ)$ $\ln[\tan(1^\circ)] + \ln[\tan(2^\circ)] + \dots + \ln[\tan(44^\circ)] + \ln[\tan(45^\circ)] + \ln[\tan(46^\circ)] + \dots + \ln[\tan(89^\circ)]$ $= \ln[\tan(1^\circ)] + \ln[\tan(2^\circ)] + \dots + \ln[\tan(44^\circ)] + \ln[\tan(45^\circ)] + \ln[\cot(44^\circ)] + \dots + \ln[\cot(1^\circ)]$ $= \ln[\tan(1^\circ)] + \ln[\tan(2^\circ)] + \dots + \ln[\tan(44^\circ)] + \ln[\tan(45^\circ)] + \ln[1/\tan(44^\circ)] + \dots + \ln[1/\tan(1^\circ)]$ $= \ln[\tan(1^\circ)] + \ln[\tan(2^\circ)] + \dots + \ln[\tan(44^\circ)] + \ln[\tan(45^\circ)] - \ln[\tan(44^\circ)] - \dots - \ln[\tan(1^\circ)]$ $= \ln[\tan(45^\circ)] = \ln[1] = 0$
16.	It will be easier to deal with $(2^x)^2 = (3x)^2 \Rightarrow 4^x = 9x^2$ since 4 is a perfect square, but realize we seek only positive x-values as the original equation has only positive solutions. Let's rewrite the last equation as $4^x - 9x^2 = 0$ and let $f(x) = 4^x - 9x^2$, now we investigate the positive roots of this function. When $x=0$, $f(x) = 1 > 0$; and when $x=0.5$, $f(x) = -0.25 < 0$. Thus, there must be a root between 0 and 0.5. When $x=3$, $f(x) = -17 < 0$; and when $x=3.5$, $f(x) = 4^{3.5} - 9 \cdot 3.5^2 = 4^3 \cdot 4^{0.5} - 9 \cdot 12.25 = 64 \cdot 2 - 110.25 > 0$. Thus, there must be a root between 3 and 3.5. The sum of a number strictly between 0 and 0.5 and one that is strictly between 3 and 3.5 is strictly between 3 and 4. Answer: C

17.	Letting n = the total number of pages, and x = the page number that was counted twice, gives: $1 + 2 + 3 + \dots + n + x = 2025$, where $1 \leq x \leq n$. Using the formula $1 + 2 + 3 + \dots + n = \frac{1}{2}n(n+1)$, which can easily be derived*, now gives: $\frac{1}{2}n(n+1) + x = 2025$ or $n(n+1) + x = 4050$. That last equation suggests that n^2 is in the vicinity of 4050. Since $60^2 = 3600$ and $70^2 = 4900$, let's try some values for n in between 60 and 70 to check for $\frac{1}{2}n(n+1) + x = 2025$. $n = 65 \Rightarrow 2145 + x = 2025$, but $1 \leq x \leq 65$, so 65 is too large for n. $n = 64 \Rightarrow 2080 + x = 2025$, but $1 \leq x \leq 64$, so 64 is too large for n. $n = 63 \Rightarrow 2016 + x = 2025$, with $1 \leq x \leq 63$, so $x = 9$. Thus, there are 63 pages, with a sum of 2016 and page number 9 was counted twice. Answer: 9 * Letting $S = 1 + 2 + 3 + \dots + n$ or $S = n + \dots + 3 + 2 + 1$ and adding these two gives: $2S = (n+1) + (n+1) + (n+1) + \dots + (n+1) \Rightarrow 2S = n(n+1) \Rightarrow S = \frac{1}{2}n(n+1)$.			
18.	ONE ON0 050 This indicates that 9 ONE ON0 050 times 0 (the 0's in the ONE ON0 050 hundreds column) plus ONE ON0 050 the 4 that carries from ONE E = 0 ON0 050 the tens column must ONE ON0 050 have a 5 in the tens ONE ON0 050 position. Only N = 6 ONE ON0 050 makes that true. Giving + ONE + ON0 + 050 $9 \cdot 6 + 4 = 58$. Hence, I = 8. ===== NINE NIN0 5150	The tens column (the N's) indicates that 9N will have an N in the units position. Thus, N = 0 or 5 since no other digit when multiplied by 9 will have that digit in the units position. $\Rightarrow$ However, N cannot be 0, as E = 0. Thus, N = 5.	050 050 050 050 050 050 050 050 + 050 ===== 5150	Thus, "ONE" = 650 and "NINE" = 5850. Answer: 8
19.	Let $y = 1 - 6x^2$, so then the original equation becomes $x = 1 - 6y^2$. Subtracting these two gives: $y - x = (1 - 6x^2) - (1 - 6y^2) \Rightarrow y - x = 6y^2 - 6x^2 \Rightarrow y - x = 6(y - x)(y + x)$ $\Rightarrow (y - x) - 6(y - x)(y + x) = 0 \Rightarrow (y - x)[1 - 6(y + x)] = 0 \Rightarrow (y - x)(1 - 6y - 6x) = 0$ $\Rightarrow \textcircled{1} y - x = 0 \text{ and } \textcircled{2} 1 - 6y - 6x = 0. \text{ Now replace } y \text{ with } 1 - 6x^2 \text{ to obtain:}$ $\textcircled{1} 1 - 6x^2 - x = 0 \Rightarrow 6x^2 + x - 1 = 0 \Rightarrow (3x - 1)(2x + 1) = 0 \Rightarrow x = \frac{1}{3}, -\frac{1}{2}.$ Since these are rational, there is no need to solve $\textcircled{2}$ (which would yield irrational roots). Hence, the sum of the rational roots is $\left(\frac{1}{3}\right) + \left(-\frac{1}{2}\right) = -\frac{1}{6}$. Answer: $-\frac{1}{6}$ <u>Alternate Solution (to obtain only the rational roots):</u> Letting $f(x) = 1 - 6x^2$, the equation can be written as $x = f(f(x))$ — which is satisfied when $f(x) = x$. Thus, we solve $1 - 6x^2 = x$ — which yields $x = \frac{1}{3}, -\frac{1}{2}$, as before.			
20.	An equivalent scenario that helps simplify the analysis is: Let's call the first person who lost his boarding pass Josh. Suppose whenever a person finds their seat is taken, they ask the person to move (to another random available seat). This is equivalent to the original problem with the exception of who is in the first 99 taken seats (i.e. the name of the person in each seat). So, by the time the last person boards, all other 98 people (not Josh) are definitely in their assigned seat. Now Josh has an equal probability of being in his assigned seat or the last person's seat. Thus, the last person's assigned seat is available with a probability of $\frac{1}{2}$. Answer: $\frac{1}{2}$			