

New York State Mathematics Association of Two-Year Colleges

Math League Contest ~ Spring 2025

Directions: You have one hour to take this test. Scrap paper is allowed. The use of calculators is NOT permitted, as well as computers, books, math tables, and notes of any kind. You are not expected to answer all the questions. However, do not spend too much time on any one problem. Four points are awarded for each correct answer, one point is deducted for each incorrect answer, and no points are awarded/deducted for blank responses. There is no partial credit. Unless otherwise indicated, answers must be given in *exact* form, i.e. in terms of fractions, radicals, π , etc.

1. A 2×2 determinant is defined as: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \equiv a \cdot d - b \cdot c$. What value for x satisfies the equation $\begin{vmatrix} 2+i & x \\ i & 2-i \end{vmatrix} = 1-i$? Where $i \equiv \sqrt{-1}$. Give the answer in $a + bi$ form.
2. At how many points do the graphs of $y = x^2$ and $y = 1.1^x$ intersect?
3. What day of the week will be 2025 days after next Monday?
A) Tuesday B) Wednesday C) Thursday D) Friday E) Saturday
4. What is the area enclosed by the graph of $|x| + |y| = 1$?
A) $\frac{1}{2}$ B) $\frac{\sqrt{2}}{2}$ C) 1 D) $\sqrt{2}$ E) 2
5. How many different triangles can be formed if one side has length 2024, another side has length 2025, and the length of the third side must be an integer?
6. If $a^2 + b^2 = 2025$, where a and b are positive integers, then what is the value of $a + b$?
Hint: 2025 is a perfect square.
7. For a two-child family, if we know that at least one of the children is a boy, then what is the probability the other child is a girl? Assume the probability of giving birth to a girl is equal to the probability of giving birth to a boy (i.e. they are both $\frac{1}{2}$).
A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) $\frac{3}{4}$

8. The sum $\sum_{n=0}^{2025} i^n = i^0 + i^1 + i^2 + i^3 + i^4 + \dots + i^{2025}$, where $i \equiv \sqrt{-1}$, simplifies to

- A) 0 B) 1 C) i D) $1-i$ E) $1+i$

9. $\sqrt{4-2\sqrt{3}}$ equals which of the following?

- A) $\sqrt{7}-2$ B) $\frac{1}{2}\sqrt{2}$ C) $\sqrt{6}-\sqrt{3}$ D) $\sqrt{3}-1$ E) $\sqrt[4]{10}-1$

10. If all 720 permutations (i.e. arrangements) of the digits 1 through 6 are listed in numerical order: $\{123456, 123465, 123546, \dots, 654321\}$, then what would be the 400th number in the list?

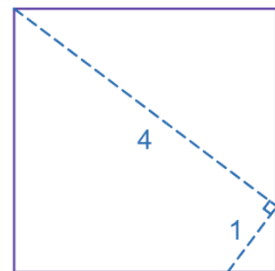
11. What value of x solves the equation $x^x = 2^{\sqrt{200}}$?

12. Which is the correct ordering of $-\log_{0.9}(10)$, $\log_{1.1}(10)$, and $\log_{(\frac{1}{0.9})}(10)$?

- A) $-\log_{0.9}(10) < \log_{(\frac{1}{0.9})}(10) < \log_{1.1}(10)$ B) $-\log_{0.9}(10) = \log_{(\frac{1}{0.9})}(10) < \log_{1.1}(10)$
 C) $-\log_{0.9}(10) = \log_{1.1}(10) < \log_{(\frac{1}{0.9})}(10)$ D) $\log_{1.1}(10) < -\log_{0.9}(10) = \log_{(\frac{1}{0.9})}(10)$
 E) $-\log_{0.9}(10) = \log_{(\frac{1}{0.9})}(10) = \log_{1.1}(10)$

13. What is the sum of all solutions, x , to the equation $(x^2 - x - 7)^{2x^2 - 5x + 1} = 1$?

14. A square has two dashed lines drawn, one with a length of 4 from the upper left corner to the right side, the other with a length of 1 perpendicular to the first line to the bottom side, as shown. What is the length of the sides of the square?

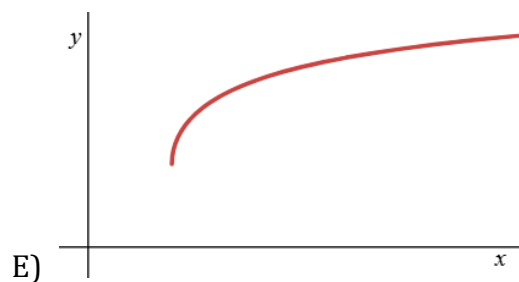
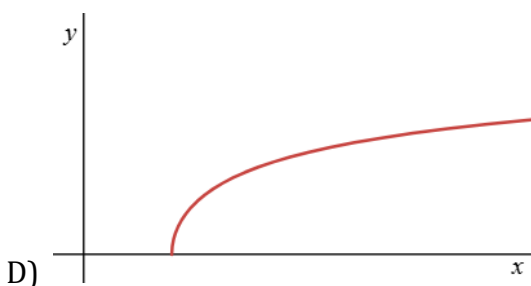
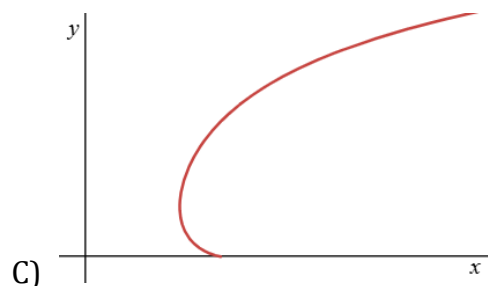
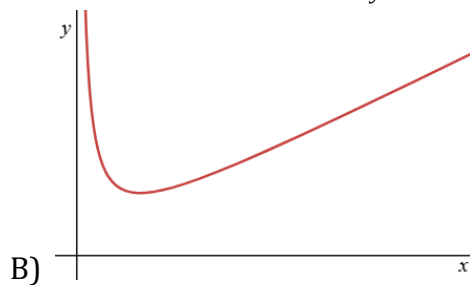
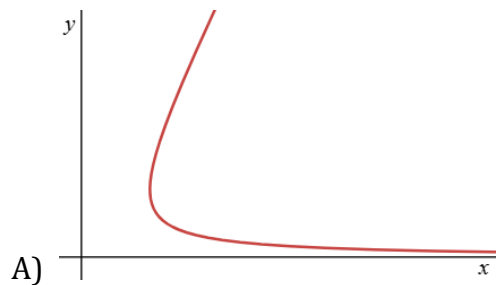


15. Which of the following statements imply that the positive real number x is irrational?

- | | |
|------------------------------|--------------------------|
| I. $\sqrt{x}$ is irrational. | A) I only. |
| II. πx is rational. | B) I and II only. |
| III. x^2 is irrational. | C) II and III only. |
| IV. x^{-1} is irrational. | D) III and IV only. |
| | E) II, III, and IV only. |

16. The equation $\log_3(\sqrt[3]{x}) = \sqrt[3]{\log_3(x)}$ has three solutions. What is the largest of the three solutions?

17. Which of the following graphs best represents the graph of $y = \log_y(x)$?



18. $\sin^2(2^\circ) + \sin^2(4^\circ) + \sin^2(6^\circ) + \dots + \sin^2(88^\circ) + \sin^2(90^\circ)$, i.e. the sum of the squares of the sine of angles with even degree measure from 2° through 90° , equals?

- A) $2\pi^2$ B) 22 C) 23 D) 23.5 E) $5 + 6\sqrt{2} + 6\sqrt{3}$

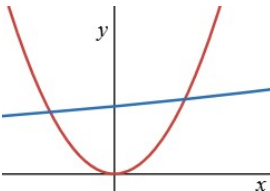
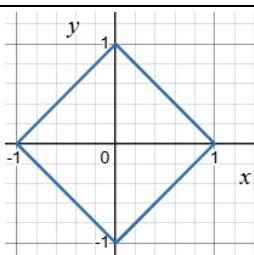
19. For what value of n is the sum

$$\sum_{k=1}^n \frac{1}{\sqrt{k} + \sqrt{k+1}} = \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{n} + \sqrt{n+1}}$$
 equal to 10?

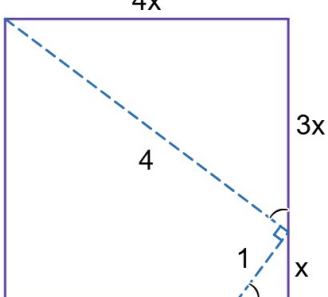
20. If the day before two days after the day before tomorrow is Monday, then what day is it today?

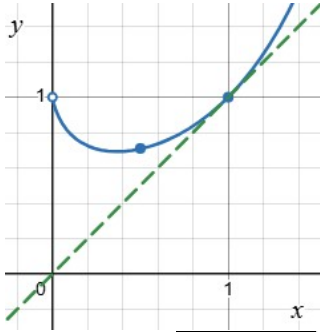
- A) Sunday B) Monday C) Tuesday D) Wednesday E) Saturday

Math League Contest ~ Spring 2025 ~ Solutions

1.	$\begin{vmatrix} 2+i & x \\ i & 2-i \end{vmatrix} = 1-i \Rightarrow (2+i)(2-i) - ix = 1-i \Rightarrow 4 - i^2 - ix = 1-i \Rightarrow 4 - (-1) - ix = 1-i$ $\Rightarrow 5 - ix = 1-i \Rightarrow ix = 4+i \Rightarrow x = \frac{4+i}{i} = 1-4i$ Answer: $1-4i$
2.	A rough sketch of the two functions clearly shows two points of intersection near the origin. However, we know that exponential <i>growth</i> (no matter how slow) will always eventually overtake any power function. Thus, there must be a third point of intersection for some “large” x-value. Answer: 3 Incidentally, that “large” x-value is nearly 96. 
3.	2025 divided by 7 is 289 with a remainder of 2. Thus, 2025 days after a Monday is 289 weeks plus 2 days. Therefore, it will be a Wednesday. Answer: B
4.	The graph of $x + y = 1$ is a square with sides length $\sqrt{2}$, as shown. Thus, the area is $\sqrt{2}^2 = 2$. Answer: E 
5.	In order for three line segments to form a triangle, the sum of the two smaller sides must be greater than the longest side. Thus, the smallest (integer) third side is of length 2, since $2 + 2024 > 2025$. Then we can have integer lengths of 3, 4, 5, ..., 2025, until the new third side becomes the longest side. Then the new side can be 2026, 2027, ..., 4048 (i.e. $2024 + 2025 - 1$). Hence, the third side may be any integer from 2 through 4048, giving 4047 different values and 4047 different triangles. Answer: 4047
6.	$2025 = 45^2 \Rightarrow a^2 + b^2 = 45^2 \Rightarrow a^2 = 45^2 - b^2 \Rightarrow a^2 = (45-b)(45+b)$ Certainly, if both $45-b$ and $45+b$ are perfect squares, then so will be their product. If $b=9$, then $45-b$ is perfect square, but $45+b$ is not. If $b=20$, then $45-b$ is perfect square, but $45+b$ is not. If $b=29$, then $45-b$ is perfect square, but $45+b$ is not. If $b=36$, then $45-b=9$ is perfect square, and so is $45+b=81$! This yields $a^2 = (45-36)(45+36) = 9 \cdot 81$. Thus, $a = \sqrt{9 \cdot 81} = 3 \cdot 9 = 27$. The wording of the problem implies there is only one possible value for the sum. Thus, either $a=27$ and $b=36$, or $a=36$ and $b=27$. In either case, $a+b=63$. Answer: 63
7.	The sample space (i.e. the set of possible outcomes) for a two-child family is $\{BB, BG, GB, GG\}$, where BG represents the first born is a boy and the second born is a girl and GB indicates the first born is a girl while the second born is a boy – with each outcome equally likely. Since we know there is at least one boy, we can eliminate the GG case, leaving only the three cases $\{BB, BG, GB\}$. Of the remaining three cases, two of them have the <i>other</i> child being a girl. Thus, the probability the <i>other</i> child is a girl is $2/3$. Answer: D

8.	$i^0 + i^1 + i^2 + i^3 + i^4 + \dots + i^{2023} + i^{2024} + i^{2025} = 1 + i + (-1) + (-i) + 1 + \dots + (-i) + 1 + i$, i.e. the powers cycle in groups of four ($n=0,1,2,3$; $n=4,5,6,7$; and so on through $n=2020,2021,2022,2023$) with each group summing to 0. Thus, the sum for $n=0$ through $n=2023$ is 0, leaving only $i^{2024} = 1$ and $i^{2025} = i$. Hence, the sum is $1 + i$. Answer: E
9.	Since all five choices can be expressed as $\sqrt{a} - \sqrt{b}$, let's start by assuming the expression can be represented as such. Thus, we start with $\sqrt{4 - 2\sqrt{3}} = \sqrt{a} - \sqrt{b}$ and then square both sides to obtain: $4 - 2\sqrt{3} = a - 2\sqrt{ab} + b$ or $4 - 2\sqrt{3} = a + b - 2\sqrt{ab}$. Equating corresponding terms gives: ① $a + b = 4$ and ② $ab = 3$. Solving these yields $a = 1$ and $b = 3$, or $a = 3$ and $b = 1$. However, $\sqrt{1} - \sqrt{3} < 0$, while $\sqrt{4 - 2\sqrt{3}} > 0$. Thus, $a = 3$ and $b = 1$. Answer: D
10.	$\underbrace{123456}_{5!=120}$ and $\underbrace{213456}_{5!=120}$ and $\underbrace{312456}_{5!=120} \Rightarrow$ There are $5! = 120$ numbers that begin with a 1, $5! = 120$ numbers that begin with a 2, and another $5! = 120$ numbers that begin with a 3. Hence, there are $3(5!) = 360$ numbers that precede 412356. We need 40 more to get to the 400th. $\underbrace{412356}_{4!=24} \Rightarrow$ There are $4! = 24$ numbers that start with 41. We need 16 more. $\underbrace{421356}_{3!=6}$ and $\underbrace{423156}_{3!=6} \Rightarrow$ There are $2(3!) = 2 \cdot 6 = 12$ numbers that start with 421 or 423. We need 4 more: 425136, 425163, 425316, and 425361. Answer: 425361
11.	We need to rewrite $2^{\sqrt{200}}$ so the base and the exponent are the same. $2^{\sqrt{200}} = 2^{\sqrt{100 \cdot 2}} = 2^{10\sqrt{2}} = 2^{\frac{1}{2} \cdot 20\sqrt{2}} = \sqrt{2}^{20\sqrt{2}} = \sqrt{2}^{5 \cdot 4\sqrt{2}} = \left(\sqrt{2}^5\right)^{4\sqrt{2}} = \left(4\sqrt{2}\right)^{4\sqrt{2}} \Rightarrow x = 4\sqrt{2}$ Answer: $x = 4\sqrt{2} = \sqrt{32} = 2^{5/2}$
12.	Let $y = -\log_{0.9}(10) = \log_{0.9}(10^{-1}) \Rightarrow 0.9^y = 10^{-1} \Rightarrow (0.9^y)^{-1} = (10^{-1})^{-1} \Rightarrow (0.9^{-1})^y = 10$ which is $\left(\frac{1}{0.9}\right)^y = 10$. Taking $\log_{(\frac{1}{0.9})}$ of both sides of the last equation gives: $y = \log_{(\frac{1}{0.9})}(10)$. Thus, $-\log_{0.9}(10) = \log_{(\frac{1}{0.9})}(10)$. Since $\frac{1}{0.9} = 1.111\dots > 1.1$, then $\log_{(\frac{1}{0.9})}(10) < \log_{1.1}(10)$. Thus, the correct ordering of the values is choice B. Answer: B

13.	$(x^2 - x - 7)^{2x^2 - 5x + 1} = 1 \Rightarrow$ Either: ① $x^2 - x - 7 = 1$ and $2x^2 - 5x + 1$ can be any value, or ② $2x^2 - 5x + 1 = 0$ and $x^2 - x - 7 \neq 0$, or ③ $x^2 - x - 7 = -1$ and $2x^2 - 5x + 1$ is even. ① $x^2 - x - 7 = 1 \Rightarrow x^2 - x - 8 = 0 \Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{33}}{2}$ ② $2x^2 - 5x + 1 = 0 \Rightarrow x = \frac{5}{4} \pm \frac{\sqrt{17}}{4}$ with $x^2 - x - 7 \neq 0$ ③ $x^2 - x - 7 = -1 \Rightarrow x^2 - x - 6 = 0 \Rightarrow x = -2, 3$. When $x = -2$, $2x^2 - 5x + 1 = 19$ odd ✗ When $x = 3$, $2x^2 - 5x + 1 = 4$ even ✓ Thus, the sum of the solutions is: $\left(\frac{1}{2} + \frac{\sqrt{33}}{2}\right) + \left(\frac{1}{2} - \frac{\sqrt{33}}{2}\right) + \left(\frac{5}{4} - \frac{\sqrt{17}}{4}\right) + \left(\frac{5}{4} + \frac{\sqrt{17}}{4}\right) + 3 = \frac{13}{2}$. Answer: $\frac{13}{2} = 6.5$
14.	 The two resulting triangles are similar, with the larger having 4 times the dimensions (as illustrated by the hypotenuses). Applying the Pythagorean Theorem to the larger triangle gives: $(3x)^2 + (4x)^2 = 4^2 \Rightarrow 25x^2 = 16 \Rightarrow x = \frac{4}{5}.$ The square has sides of length $4x = 4\left(\frac{4}{5}\right) = \frac{16}{5}$. Answer: $\frac{16}{5} = 3.2$
15.	I. $\sqrt{x}$ being irrational does not guarantee x is irrational (e.g. $\sqrt{2}$ is irrational, but 2 is rational). ✗ II. πx being rational does imply that x cannot be rational, since the product of an irrational number (π) and a non-zero rational number is irrational. ✓ III. x^2 being irrational does imply that x is irrational, since the square of a rational number will be rational. ✓ IV. x^{-1} being irrational does imply that x is irrational, since the reciprocal of a non-zero rational number will be rational. ✓ Answer: E
16.	$\log_3(\sqrt[3]{x}) = \sqrt[3]{\log_3(x)} \Rightarrow \log_3(x^{1/3}) = \sqrt[3]{\log_3(x)} \Rightarrow \frac{1}{3}\log_3(x) = \sqrt[3]{\log_3(x)}$, now cube both sides to obtain: $\frac{1}{27}\log_3^3(x) = \log_3(x) \Rightarrow \log_3^3(x) - 27\log_3(x) = 0 \Rightarrow \log_3(x)(\log_3^2(x) - 27) = 0$ $\Rightarrow \log_3(x) = 0$ or $\log_3^2(x) - 27 = 0 \Rightarrow \boxed{x=1}$ or $\log_3(x) = \pm\sqrt{27} \Rightarrow \boxed{x=3^{-\sqrt{27}}}$ or $\boxed{x=3^{\sqrt{27}}}$ With $x = 3^{\sqrt{27}} = 3^{3\sqrt{3}} = 27^{\sqrt{3}}$ the largest of the three solutions. Answer: $3^{\sqrt{27}} = 3^{3\sqrt{3}} = 27^{\sqrt{3}}$

17.	$y = \log_y(x) \Rightarrow x = y^y$, which is the inverse function of $y = x^x$. Thus, the graph of $y = \log_y(x)$ can be obtained by reflecting the graph of $y = x^x$ about the line $y = x$. Plotting some select points of $y = x^x$ will give us an idea of its graph. As $x \rightarrow 0$, $x^x \rightarrow 1$; when $x = 0.5$, $y = 0.5^{0.5} = \sqrt{1/2} = \sqrt{2}/2 \approx 0.7$; when $x = 1$, $y = 1$; then for $x > 1$ the graph increases faster and faster (i.e. concave up). The reflection of the graph shown about the line $y = x$ (dashed line) is best represented by choice C. 	Answer: C
18.	Since the sine and cosine of complementary angles are equal, i.e. $\sin(x^\circ) = \cos(90^\circ - x^\circ)$, we can replace the sine of all the angles from 46° through 88° with the cosine of the complementary angles and $\sin(90^\circ)$ with 1: $\sin^2(2^\circ) + \sin^2(4^\circ) + \dots + \sin^2(42^\circ) + \sin^2(44^\circ) + \cos^2(44^\circ) + \cos^2(42^\circ) + \dots + \cos^2(4^\circ) + \cos^2(2^\circ) + 1$ Now group them to make use of the Pythagorean identity (22 times): $\underbrace{\sin^2(2^\circ) + \cos^2(2^\circ)}_1 + \underbrace{\sin^2(4^\circ) + \cos^2(4^\circ)}_1 + \dots + \underbrace{\sin^2(42^\circ) + \cos^2(42^\circ)}_1 + \underbrace{\sin^2(44^\circ) + \cos^2(44^\circ)}_1 + 1$ $= \underbrace{1 + 1 + \dots + 1 + 1 + 1}_{22} = 23$	Answer: C
19.	Multiply each term by the conjugate of the denominator over itself, i.e. by $\frac{\sqrt{k} - \sqrt{k+1}}{\sqrt{k} - \sqrt{k+1}}$: $\Rightarrow \sum_{k=1}^n \frac{1}{\sqrt{k} + \sqrt{k+1}} \cdot \frac{\sqrt{k} - \sqrt{k+1}}{\sqrt{k} - \sqrt{k+1}} = \sum_{k=1}^n \frac{\sqrt{k} - \sqrt{k+1}}{k - (k+1)} = \sum_{k=1}^n \frac{\sqrt{k} - \sqrt{k+1}}{-1} = \sum_{k=1}^n (\sqrt{k+1} - \sqrt{k})$ $= (\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + (\sqrt{5} - \sqrt{4}) + \dots + (\sqrt{n} - \sqrt{n-1}) + (\sqrt{n+1} - \sqrt{n})$ $= -\sqrt{1} + \sqrt{n+1} = \sqrt{n+1} - 1$ Now solve $\sqrt{n+1} - 1 = 10$ for n: $\sqrt{n+1} = 10 + 1 \Rightarrow n+1 = 11^2 \Rightarrow n = 121 - 1 = 120$	Answer: 120
20.	Let's call today x, so tomorrow is $x + 1$. The day before tomorrow is today, x. Two days after the day before tomorrow is $x + 2$. The day before two days after the day before tomorrow is $x + 1$, which is tomorrow. So, if tomorrow is Monday, then today is Sunday.	Answer: A